Math 10B with Professor Stankova
Worksheet, Discussion #23; Thursday, 4/18/2019
GSI name: Roy Zhao

Maximum Likelihood Estimation (MLE)

Concepts

1. The **likelihood function** $L(\theta)$ which is the probability that we see the data we see if we set the parameter equal to θ. Namely, $L(\theta|x_1, \ldots, x_n) = P(x_1, \ldots, x_n|\theta)$, the probability we see $x_1, \ldots, x_n$ if our parameter is equal to θ. Then we choose the value of θ that maximizes this function by taking the derivative and setting it equal to 0.

Distribution	PMF/PDF	$E(X)$	Variance
Uniform	If $\#R(X) = n$, then $f(x) = \frac{1}{n}$ for all $x \in R(X)$.	$\sum_{i=1}^n \frac{x_i}{n}$	$\sum_{i=1}^n \frac{(x_i-\mu)^2}{n}$
Bernoulli Trial	$f(0) = 1-p$, $f(1) = p$	p	$Var(X) = p(1-p)$
Binomial	$f(k) = \binom{n}{k}p^k(1-p)^{n-k}$	np	$np(1-p)$
Geometric	$f(k) = (1-p)^k p$	$\frac{1-p}{p}$	$Var(X) = \frac{1-p}{p^2}$
Hyper-Geometric	$f(k) = \frac{\binom{m}{k}\binom{N-m}{n-k}}{\binom{N}{n}}$	$\frac{nm}{N}$	$\frac{nm(N-m)(N-n)}{N^2(N-1)}$
Poisson	$f(k) = \frac{\lambda^k e^{-\lambda}}{k!}$	λ	λ
Pareto	$f(x) = \frac{\alpha-1}{x^\alpha}, x \geq 1$	$\frac{\alpha-1}{\alpha-2}$	$\frac{\alpha-1}{(\alpha-2)^2(\alpha-3)}$
Uniform	$f(x) = \frac{1}{b-a}$ for $a \leq x \leq b$	$\frac{a+b}{2}$	$\frac{(b-a)^2}{12}$
Normal	$f(x) = \frac{1}{\sigma\sqrt{2\pi}}e^{-(x-\mu)^2/(2\sigma^2)}$	μ	σ^2
Exponential	$f(x) = ce^{-cx}$	$\frac{1}{c}$	$\frac{1}{c^2}$
Laplacian	$f(x) = \frac{1}{2}e^{-\|x\|}$	0	2

Examples

2. The number of threes made during an NBA game is Poisson distributed. Last Saturday, the number of threes made were $14, 26, 25$, and 13. Calculate the maximum likelihood estimate for the parameter λ.

Problems

3. True False The maximum likelihood estimate for the standard deviation of a normal distribution is the sample standard deviation ($\hat{\sigma} = s$).

4. True False The maximum likelihood estimate is always unbiased.

5. You have a coin that you think is biased. you flip it 4 times and get the sequence $HHHT$. What is the maximum likelihood estimate for the probability of getting heads?

6. During Cal Day, my two friends and I asked prospective students where they were from until we found someone who wasn't from California. We had to ask $23, 18, 46$ people respectively before finding someone not from California. What is the maximum likelihood estimate for the percentage of students from California?

7. You know that baby weights are normally distributed with mean μ and standard deviation σ. You have three babies weighing $7, 8, 9$ ounces. What is the maximum likelihood for μ?

Hypothesis Testing

Concepts

8. In general, statistics does not allow you to prove anything is true, but instead allows you to show that things are probably false. So when we do hypothesis testing, the **null hypothesis** H_0 is something that we want to show is false and the **alternative hypothesis** H_1 is something that you want to show is true. For example, to show that a drug cures cancer, the null hypothesis would be that the drug does nothing and the alternative hypothesis would be that the drug does help cure cancer.

   A **type 1 error** is rejecting a true null which means that in our example, saying a drug cures cancer when it doesn't. A **type 2 error** is failing to reject a false null which means in our case as saying that the drug doesn't do anything when it does. The **significance level** is the probability of making a type 1 error. The **power** is 1 minus the probability of making a type 2 error.

Examples

9. Chip bag manufacturers claim the weights are normally distributed with a mean of 14 ounces and a standard deviation of 0.5 ounces. Your bag is 13 ounces. What can you say with significance level $\alpha = 0.05$?

Problems

10. True False The null hypothesis is something we want to be false.

11. True False If we get a p value that is not smaller than α, then we have shown that the null hypothesis is true.

12. True False We want our test to have a high significance level and high power.

13. True False A type-2 error made by a road patrol may result in letting drunken drivers continue driving.

14. You think a die is rigged so that it will roll 5 less often. You continually roll the die until you get a 5 and you have to roll 10 times before this happens. Is your suspicion correct with $\alpha = 0.05$?

15. (True story) A woman claims that she can smell when someone has Parkinson's disease. She is given 10 people's shirts and correctly said whether the person had the disease in 9 of the 10 cases. Does she have this ability with $\alpha = 0.05$? (The 10th person who she said had Parkinson's actually developed it months later so she was really 10 for 10).

16. (True story) A woman claimed that she could tell whether milk or tea was added first to a cup. She was given 4 cups with milk added first then tea and 4 cups of the opposite. She guessed all 8 correctly. Can we say that she has this ability with $alpha = 0.05$?

17. If the woman in the previous problem had only been given 4 total cups to test, explain why we would never be able to reject the null hypothesis.

18. Some scientists publish a study that says that the average height of men is normally distributed with mean 64 inches and standard deviation 2 inches. If I am 67 inches tall, can I say that the scientists are wrong with $\alpha = 0.05$?